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Stacking Sequence Optimization of Simply Supported Laminates with Stability and Strain Constraints

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An integer programming formulation for the design of symmetric and balanced rectangular composite laminates with simply supported boundary conditions subject to buckling and strain constraints is presented. The design variables that define the stacking sequence of the laminate are ply-identity zero-one integers. The buckling constraint is linear in terms of the ply-identity design variables, but strains are nonlinear functions of these variables. A linear approximation is developed for the strain constraints so that the problem can be solved by sequential linearization using the branch and bound algorithm. Examples of graphite-epoxy plates under biaxial compression are presented. Optimum stacking sequences obtained using the linear approximation are compared with global optimum designs obtained using a genetic search procedure.

Introduction

THE growing use of high-performance composite materials has stimulated active interest in the development of optimization procedures for the design of laminates. Due to their high strength, composite plates are usually thin laminates that are buckling critical. Thus optimization of composite plates to maximize buckling loads has drawn considerable attention in recent years.[1-7] Typical design variables are ply orientations or the thicknesses of plies with prescribed orientations. In many practical applications the ply orientations are limited to 0, 90, and ± 45 deg, and the thicknesses can only be integer multiples of the lamina thickness. Thus the basic design problem is to determine the stacking sequence of the composite laminate.

The optimization of the laminate stacking sequence is an integer programming problem. Traditionally, the design variables of the laminate stacking sequence design problem were selected to be the number of plies of a given orientation.[8-11] Although ply-thickness variables lead to a nonlinear problem, Haftka and Walsh[12] have recently shown that the buckling optimization of laminated plates with zero-one ply-identity design variables results in a linear optimization problem.

The objective of the present work is to extend the formulation of Ref. 12 to include strain constraints. Strains are nonlinear functions of both ply-thickness and ply-identity variables, so that a linear approximation is developed for the strains. A maximum strain criterion[13] is used to predict the laminate failure due to in-plane compressive loads.

Problem Formulation

A simply supported laminated composite plate of dimensions $a \times b$ (see Fig. 1) subjected to in-plane compressive loads λN_x and λN_y is considered. The failure load parameter λ_c is the critical value of λ for which the laminate buckles or exceeds the allowable strain. The laminate is made up of N plies with orientations restricted to ± 45, 0, 90 deg plies. The orientation of the ith ply is identified by zero-one ply-identity design variables o_i, n_i, f_i^p, and f_i^m, respectively. If the ith ply is a 0 deg ply, then o_i is equal to 1, otherwise o_i is equal to 0. Similar definitions link the other three integer design variables to the 90, 45, and −45 deg plies, respectively. The vector of design variables is denoted as v. The composite laminate is assumed to be symmetric and balanced (i.e., the number of 45 deg plies is equal to the number of −45 deg plies). Each ply has a constant thickness t (equal to the nominal ply thickness). Thus the total thickness of the composite laminate is $h = Nt$. The laminate is optimized to maximize failure load λ_c.

Buckling Analysis

An orthotropic composite laminate under biaxial loads buckles when the load parameter λ reaches a critical value

$$\lambda_{cr}(m,n) = \pi^2 \frac{[D_{11}(m/a)^4 + 2(D_{12} + 2D_{66})(m/a)^2(n/b)^2 + D_{22}(n/b)^4]}{(m/a)^2 N_x + (n/b)^2 N_y} \tag{1}$$

where m and n are the number of half-waves in the x and y directions, respectively, selected so as to minimize λ_{cr}. In the present analysis the minimization over m and n is performed by checking for all values of m and n between 1 and m_f and n_f respectively, where m_f and n_f are preselected numbers that cover the expected number of half-waves. The flexural stiffnesses D_{11}, D_{12}, D_{22}, and D_{66} can be expressed in terms of three integrals: V_{0D}, V_{1D}, and V_{3D}, which depend on the stacking sequence[14] and five material invariants U_i, $i = 1, \ldots, 5$, as

$$\begin{aligned} D_{11} &= U_1 V_{0D} + U_2 V_{1D} + U_3 V_{3D} \\ D_{22} &= U_1 V_{0D} - U_2 V_{1D} + U_3 V_{3D} \\ D_{12} &= U_4 V_{0D} - U_3 V_{3D} \\ D_{66} &= U_5 V_{0D} - U_3 V_{3D} \end{aligned} \tag{2}$$

The integrals V_{0D}, V_{1D}, and V_{3D} are given as[12]

$$V_{0D} = \int_{-h/2}^{h/2} z^2 \, dz = \frac{2t^3}{3} \sum_{k=1}^{N/2} [k^3 - (k-1)^3](o_k + n_k + f_k^p + f_k^m)$$

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$$
\begin{aligned}
V_{1D} &= \int_{-h/2}^{h/2} z^2 \cos 2\theta \, dz \\
&= \frac{2t^3}{3} \sum_{k=1}^{N/2} [k^3 - (k-1)^3](o_k - n_k) \\
V_{3D} &= \int_{-h/2}^{h/2} z^2 \cos 4\theta \, dz \\
&= \frac{2t^3}{3} \sum_{k=1}^{N/2} [k^3 - (k-1)^3](o_k + n_k - f_k^p - f_k^m)
\end{aligned}
\tag{3}
$$

In Eq. (3), h is the total thickness of the composite laminate, z is the distance from the plane of symmetry (see Fig. 1), θ is the ply orientation angle, and the symmetry of the laminate is taken into consideration. Unlike conventional practice, it is more convenient to number the plies so that the first ply ($k = 1$) is nearest to the plane of symmetry of the composite laminate, and the last ply is on the outside ($k = N/2$). The material invariants are

$$
\begin{aligned}
U_1 &= \tfrac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}) \\
U_2 &= \tfrac{1}{2}(Q_{11} - Q_{22}) \\
U_3 &= \tfrac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}) \\
U_4 &= \tfrac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}) \\
U_5 &= \tfrac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})
\end{aligned}
\tag{4}
$$

where the reduced stiffnesses Q_{ij} are given in terms of the engineering constants as shown in Ref. 13.

Strain Calculation

In the absence of applied shear loads, the laminate strains ϵ_x and ϵ_y can be calculated (for a load factor $\lambda = 1$) from

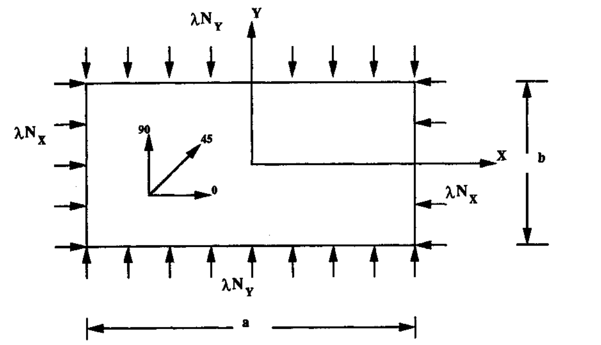
$$
\begin{aligned}
\epsilon_x &= \frac{(A_{22}N_x - A_{12}N_y)}{(A_{11}A_{22} - A_{12}^2)} \\
\epsilon_y &= \frac{(A_{11}N_y - A_{12}N_x)}{(A_{11}A_{22} - A_{12}^2)}
\end{aligned}
\tag{5}
$$

where N_x and N_y are uniform across the plate and equal to the applied edge forces per unit length. The strains for the kth ply may be calculated from the transformation

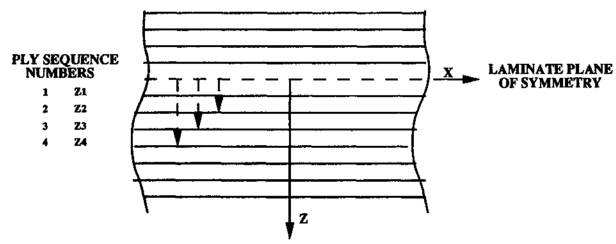
$$
\begin{aligned}
\epsilon_1^k &= \cos^2\theta_k \epsilon_x + \sin^2\theta_k \epsilon_y \\
\epsilon_2^k &= \sin^2\theta_k \epsilon_x + \cos^2\theta_k \epsilon_y \\
\gamma_{12}^k &= \sin 2\theta_k (\epsilon_y - \epsilon_x)
\end{aligned}
\tag{6}
$$

The coefficients A_{11}, A_{12}, and A_{22} are linear functions of the design variables and can be written in terms of through-the-thickness integrals and material invariants U_i as

$$
\begin{aligned}
A_{11} &= U_1 V_{0A} + U_2 V_{1A} + U_3 V_{3A} \\
A_{22} &= U_1 V_{0A} - U_2 V_{1A} + U_3 V_{3A} \\
A_{12} &= U_4 V_{0A} - U_3 V_{3A} \\
A_{66} &= U_5 V_{0A} - U_3 V_{3A}
\end{aligned}
\tag{7}
$$



a)



b)

Fig. 1 Laminate geometry and applied loading: a) laminate plate geometry and applied loading, b) ply sequence numbering and location.

The integrals V_{0A}, V_{1A}, and V_{3A} are given in terms of the ply identity variables

$$
\begin{aligned}
V_{0A} &= \int_{-h/2}^{h/2} dz \\
&= 2t \sum_{k=1}^{N/2} (o_k + n_k + f_k^p + f_k^m) \\
V_{1A} &= \int_{-h/2}^{h/2} \cos 2\theta \, dz \\
&= 2t \sum_{k=1}^{N/2} (o_k - n_k) \\
V_{3A} &= \int_{-h/2}^{h/2} \cos 4\theta \, dz \\
&= 2t \sum_{k=1}^{N/2} (o_k + n_k - f_k^p - f_k^m)
\end{aligned}
\tag{8}
$$

Even though the extensional stiffnesses A_{ij} are linear functions of the design variables, the strains calculated by Eq. (5) are nonlinear functions of these variables. These strains are linearized by a linear Taylor series in A_{ij}.

Strain Approximation

The strains approximated by a Taylor's series expansion in terms of the extensional stiffnesses are

$$
\epsilon_L(v) = \epsilon(v_0) + \left(\frac{\partial \epsilon}{\partial A_{11}}\right)_{v_0} (A_{11} - A_{12}^{v_0}) + \left(\frac{\partial \epsilon}{\partial A_{12}}\right)_{v_0} (A_{12} - A_{12}^{v_0}) + \left(\frac{\partial \epsilon}{\partial A_{22}}\right)_{v_0} (A_{22} - A_{22}^{v_0})
\tag{9}
$$

where ϵ is a typical strain component ($\lambda = 1$), ϵ_L is its linear approximation, and $A_{ij}^{v_0}$ and A_{ij} are the extensional stiffnesses calculated at the nominal design point v_0 and neighboring designs, respectively. The derivatives of the strain with respect to the extensional stiffnesses at the nominal design point are calculated in terms of the midplane strains and the extensional stiffnesses at the nominal design. The linear strain approximation can thus be constructed along a particular fiber orientation and transverse to it by evaluating the strains ϵ_1^k, ϵ_2^k, and γ_{12}^k for each orientation (since the orientation is chosen a priori, either 0 or 45 deg) in terms of the midplane strains using Eq. (6). For example, the strains along and transverse to the 45 deg fibers and in shear can be derived as

$$\begin{aligned} \epsilon_1 = \epsilon_2 &= \frac{1}{2}(\epsilon_x + \epsilon_y + \gamma_{xy}) \\ &= \frac{1}{2}\left[\frac{(A_{22}-A_{12})N_x + (A_{11}-A_{12})N_y}{(A_{11}A_{22}-A_{12}^2)}\right] \\ \gamma_{12} &= (\epsilon_y - \epsilon_x) \\ &= \left[\frac{(A_{11}+A_{12})N_y - (A_{22}+A_{12})N_x}{(A_{11}A_{22}-A_{12}^2)}\right] \end{aligned} \tag{10}$$

The derivatives needed for Eq. (9) can be obtained by differentiating Eq. (10). For example, the derivative of the strain along the 45 deg fiber with respect to A_{11} can be written as

$$\left(\frac{\partial \epsilon_1}{\partial A_{11}}\right) = \frac{1}{2}\left[\frac{(A_{12}-A_{22})}{(A_{11}A_{22}-A_{12}^2)}\right]\epsilon_x \tag{11}$$

Similar strain derivatives with respect to A_{22} and A_{12} were derived. The extensional stiffnesses are a linear function of the ply-identity design variables; thus the strain approximation is a linear function of the ply-identity variables. It is also important to note that the strains are initially calculated based on some reference value of the load. To implement the strain constraint, they have to be multiplied by the value of the load multiplier λ_c.

Optimization Formulation

The optimization problem is to maximize the failure load λ_c of the laminate, where failure denotes either buckling or excessive strains. For a large number of plies (N), the number of design variables can become large and unwieldy. Herein, this problem is alleviated by considering the laminate to be made up of stacks of two plies of the same orientation (for 0 and 90 deg plies) in a stack. The balanced condition can be taken into account by stacking a 45 deg ply and a −45 deg ply together in a stack. The ply-identity variables are modified into stack-identity variables. The stack-identity design variable for a ±45 deg stack is denoted f_j, which is equal to one if the jth stack is a ±45 deg stack and is equal to zero otherwise. Similarly, o_j is one if the jth stack is 0_2 deg and is zero otherwise. Using two ply stacks reduces the constraints by implicitly enforcing the balanced condition. The optimization problem can be stated for integer stack-identity variables as follows:

Find λ_c and o_j, n_j and f_j,

$$\forall \quad j = 1, \ldots, N/4$$

to maximize λ_c such that

$$\begin{aligned} \lambda_c \le \lambda_{cr}(m,n), \quad &\forall \quad m = 1, \ldots, m_f \\ &\forall \quad n = 1, \ldots, n_f \\ o_j + n_j + f_j = 1, \quad &\forall \quad j = 1, \ldots, N/4 \\ \lambda_c \epsilon_i \le \epsilon_i^{ua}/f, \quad &\forall \quad i = 1, \ldots, n_\epsilon \end{aligned} \tag{12}$$

where ϵ_i^{ua} denotes the strain limit, f is a safety factor, and n_ϵ is the number of strain constraints. The last constraint of problem (12) is nonlinear. Linearizing the strain components ϵ_i is not sufficient to linearize the problem because of the presence of the multiplier λ_c. Instead, this constraint can be replaced by

$$\epsilon_i \le \epsilon_i^{ua}/f\lambda_c \tag{13}$$

The failure load factor λ_c was expanded about the nominal design using a Taylor's series expansion resulting in

$$\epsilon_i \le \frac{\epsilon_i^{ua}}{f}\left(\frac{1}{\lambda_0} - \frac{\lambda_c - \lambda_0}{\lambda_0^2}\right) \tag{14}$$

where λ_o is the failure load factor of the nominal design. This can be simplified as

$$\lambda_0 \epsilon_i + \left(\frac{\lambda_c}{\lambda_0}\right)\frac{\epsilon_i^{ua}}{f} \le 2\,\frac{\epsilon_i^{ua}}{f} \tag{15}$$

Finally ϵ_i in Eq. (15) is linearized using the strain approximation, Eq. (9).

Results

Results were obtained for graphite-epoxy plates [$E_1 = 18.5 \times 10^6$ psi (127.57 GPa); $E_2 = 1.89 \times 10^6$ psi (13.03 GPa); $G_{12} = 0.93 \times 10^6$ psi (6.41 GPa); $t_{ply} = 0.005$ in. (0.0127 cm); $\nu_{12} = 0.3$; and the ultimate strains $\epsilon_1^{ua} = 0.008$; $\epsilon_2^{ua} = 0.029$; $\gamma_{12}^{ua} = 0.015$]. The computations were performed with the LINDO program[15] that employs the branch-and-bound algorithm. Initially, biaxial loading was applied to a laminated plate of aspect ratio $(a/b) = 4$ (see Fig. 1), and the buckling load was maximized for various ratios of the applied load.

Buckling Critical Designs

Some of the optimum designs obtained for different N_y/N_x ratios were buckling critical designs that had four or more contiguous plies of the same orientation. When the number of contiguous plies of the same orientation are large, composite laminates are known to experience matrix cracking. Therefore, it is desirable to limit the number of such contiguous plies. (Although such a limit helps alleviate the problem of matrix cracking, we do not imply that it is guaranteed to prevent the problem.) In the present formulation (using stacks of two plies for modeling the stacking sequence), only stacks containing 0 and 90 deg exceeding the four contiguous ply constraint would have to be taken into account. For the ±45 deg plies the condition is satisfied automatically.

To demonstrate the ease of the implementation of the four contiguous ply constraint, the buckling critical design for a load ratio of $N_y/N_x = 0.5$ was chosen. The buckling critical design had 12 contiguous 90 deg plies (i.e., six stacks). The linear constraint was implemented in terms of the stack-identity variables by adding the condition (for example, for stack locations 3 to 5).

$$n_3 + n_4 + n_5 \le 2 \tag{16}$$

to LINDO. Other stack locations were handled similarly. At the midplane however, due to the symmetry condition across the midplane, the constraint must be written as

$$n_1 + n_2 \le 1 \tag{17}$$

Buckling critical designs with and without the four contiguous ply constraint are compared in Fig. 2 for a load ratio of $N_y/N_x = 0.5$. The figure shows a) the optimum design without contiguous ply constraint, b) a near optimum design with only six contiguous 90 deg plies, and c) the design obtained with the constraint.

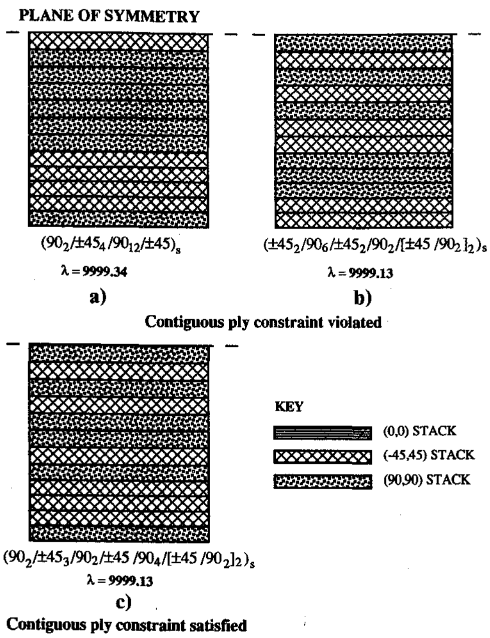


Fig. 2 Buckling critical designs with four contiguous ply constraint for $N_y = 0.5$ lb/in. (48 plies, $a = 20$ in., $N_x = 1$ lb/in.).

The contiguous ply constraint reduced the buckling load by only a small fraction of one percent.

Stability and Strain Failure

To study the design for buckling and strain constraints, we considered only designs where a buckling critical design would violate the strain constraints. All buckling critical designs for load ratios $N_y/N_x < 0.75$ fell into this category. The search for an optimum design satisfying stability as well as strain constraints was initiated by considering a buckling critical design that satisfies the four contiguous ply constraint.

The optimization procedure was started by linearizing the strain constraints about the buckling-only design and using LINDO to optimize the approximate problem. The procedure was then repeated by linearizing the strains about the approximate optimum. Each approximate optimization is called herein as a design cycle. The sequential linearization procedure converged to an optimum design within a few design cycles (usually 3–5 cycles).

Close to the optimum, oscillations between designs in the vicinity of the optimum that satisfied all constraints (but were not optimum) or designs that violated at least one constraint were observed. This oscillatory behavior was attributed to the nature of the strain approximation. Such oscillations are usually handled by move limits as bounds on the design variables. Since the stack-identity variables were discrete (0–1) variables, it was not feasible to apply move limits to the stack-identity variables. Hence move limits were applied as bounds on the extensional stiffnesses A_{ij} as

$$
\begin{aligned}
A_{11}^L &\le A_{11} \le A_{11}^U \\
A_{12}^L &\le A_{12} \le A_{12}^U \qquad (18) \\
A_{22}^L &\le A_{22} \le A_{22}^U
\end{aligned}
$$

The move limits were initially 15% and were reduced as the solutions converged. The final optimum designs obtained from LINDO were compared with the global optimum design obtained using a genetic algorithm[16–18] and exact nonlinear expressions for strains.

Buckling critical designs satisfying the four contiguous ply constraint are compared with the designs satisfying stability

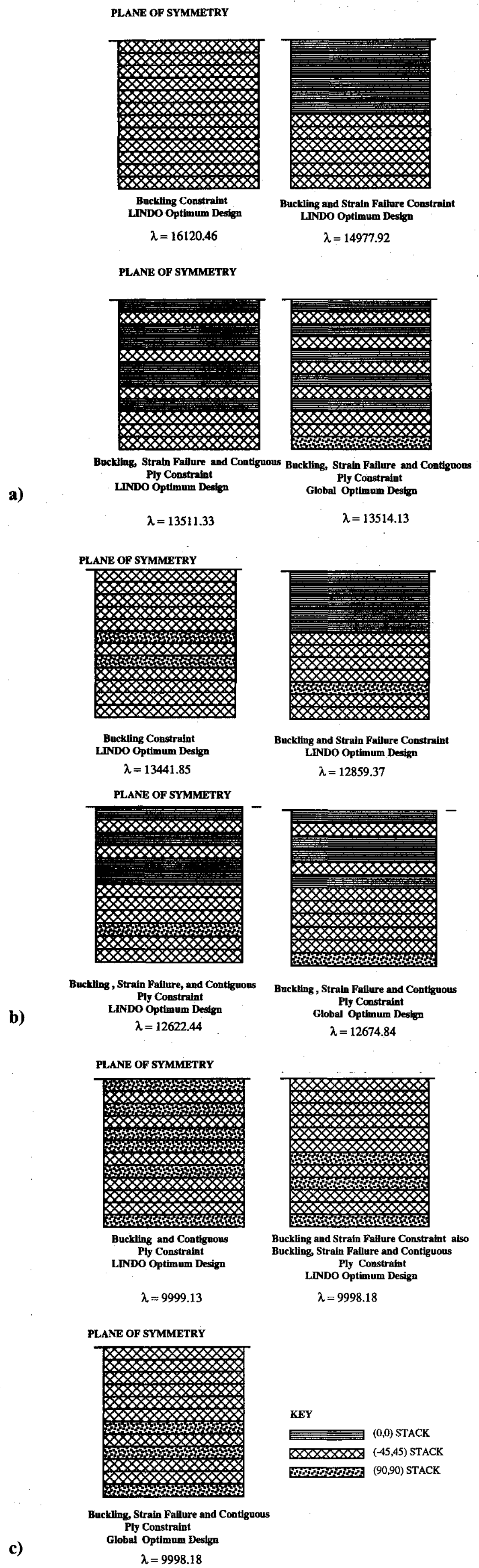


Fig. 3 Maximum buckling load results for a graphite-epoxy laminated plate (48 plies, $a = 20$ in., $N_x = 1$ lb/in.): a) $N_y = 0.125$ lb/in., b) $N_y = 0.25$ lb/in., c) $N_y = 0.5$ lb/in.

Table 1 Comparison of optimum designs

Applied constraints[a]	Stacking sequence	Failure load factor, λ_c	Optimization algorithm	Strains along 0 deg fibers	Strains along 45 deg fibers	Strains
		Load case: $N_x=1.0$, $N_y=0.125$				
B	$([\pm 45]_{12})_s$	16,120.46	Branch and bound	0.01928	0.00348	ϵ_{11}
				−0.01232	0.00348	ϵ_{22}
				0.00000	−0.03160	γ_{12}
B,S	$(\pm 45_6, 0_{12})_s$	14,977.92	Branch and bound+ linearization	0.00525	0.00196	ϵ_{11}
				−0.00134	0.00196	ϵ_{22}
				0.00000	−0.00659	γ_{12}
B,S,C	$(\pm 45_3, 0_2, [\pm 45, 0_4]_2, \pm 45, 0_2)_s$	13,511.33	Branch and bound+ linearization	0.00475	0.00177	ϵ_{11}
				−0.00121	0.00177	ϵ_{22}
				0.00000	−0.00595	γ_{12}
B,S,C	$(90_2, \pm 45_2, 0_2, [\pm 45, 0_2]_4)_s$	13,514.13	Genetic algorithm	0.00533	0.00241	ϵ_{11}
				−0.00177	0.00241	ϵ_{22}
				0.00000	−0.00754	γ_{12}
		Load case: $N_x=1.0$, $N_y=0.25$				
B	$(\pm 45_4, 90_2, \pm 45, 90_2, \pm 45_5)_s$	13,441.85	Branch and bound	0.01385	0.00581	ϵ_{11}
				−0.00223	0.00581	ϵ_{22}
				0.00000	−0.01609	γ_{12}
B,S	$(\pm 45_2, 90_2, \pm 45_4, 0_{10})_s$	12,859.37	Branch and bound+ linearization	0.00476	0.00250	ϵ_{11}
				−0.00237	0.00250	ϵ_{22}
				0.00000	−0.00453	γ_{12}
B,S,C	$(\pm 45_2, 90_2, \pm 45_3, 0_4, [\pm 45, 0_2]_2)_s$	12,622.44	Branch and bound+ linearization	0.00531	0.00248	ϵ_{11}
				−0.00035	0.00248	ϵ_{22}
				0.00000	−0.00566	γ_{12}
B,S,C	$(90_2, \pm 45_5, 0_2, \pm 45, 0_4, \pm 45, 0_2)_s$	12,674.84	Genetic algorithm	0.00533	0.00258	ϵ_{11}
				−0.00035	0.00258	ϵ_{22}
				0.00000	−0.00590	γ_{12}
		Load case: $N_x=1.0$, $N_y=0.5$				
B,C	$(90_2, \pm 45_3, 90_2, \pm 45, 90_4, [\pm 45, 90_2]_2)_s$	9,999.13	Branch and bound		0.00496	ϵ_{11}
					0.00496	ϵ_{22}
					−0.01077	γ_{12}
B	$(90_2, \pm 45_4, 90_{12}, \pm 45)_s$	9,999.34	Branch and bound+ linearization		0.00547	ϵ_{11}
					0.00547	ϵ_{22}
					−0.01154	γ_{12}
B,S,C B,S	$(90_2, \pm 45_2, 90_2, \pm 45, 90_2, \pm 45_6)_s$	9,998.18	Branch and bound+ linearization		0.00380	ϵ_{11}
					0.00380	ϵ_{22}
					−0.00962	γ_{12}
B,S,C	$(90_2, \pm 45_2, 90_2, \pm 45, 90_2, \pm 45_6)_s$	9,998.18	Genetic algorithm		0.00380	ϵ_{11}
					0.00380	ϵ_{22}
					−0.00962	γ_{12}

[a]Applied constraints: B—buckling, S—strain failure, C—contiguous ply.

and strain failure constraints with and without the contiguous ply constraints obtained from LINDO and with global optima obtained by the genetic algorithm in Fig. 3. The strains in these designs are given in Table 1. It is observed that, for load ratios of 0.125 and 0.25, 0 deg fibers were added to reduce the high axial strains. The failure load factor decreased by 16% for $N_y=0.125$ and by 6.05% for $N_y=0.25$. The design obtained using LINDO was inferior to the global optimum by less than 0.5%. This can be attributed to the use of the strain approximation in LINDO. For the load ratio of 0.5, the violation in shear strain for the buckling only optimal design was only 7% and LINDO converged to the global optimum (see Table 1).

The effect of the four contiguous ply constraint is also shown in Fig. 3 and Table 1. Without the strain constraints, the contiguous ply constraint had a negligible effect in the case of load ratio of $N_y/N_x=0.5$ and no effect for load ratios of $N_y/N_x=0.125$ and 0.25.

With the strain constraint and in the absence of the four contiguous ply constraint, the layers close to the midplane were replaced by 0 deg plies. For the load ratio of 0.125, the design has 24 contiguous 0 deg plies, whereas for the load ratio of 0.25 there are 20 contiguous 0 deg plies. The contiguous ply constraint resulted in a loss of 9.8 and 1.85% in failure load for the load ratios of 0.125 and 0.25, respectively. For a load ratio of $N_y/N_x=0.5$, the design without the contiguous ply constraint satisfied this constraint anyhow.

Concluding Remarks

An integer programming formulation based on ply-identity design variables for the design of symmetric and balanced composite laminates subject to stability and strain constraints was presented. The buckling constraint was linear, but the strains were nonlinear functions of the design variables. A strain approximation that is linear in terms of the ply-identity design variables was proposed and incorporated in a sequential approximate optimization procedure. Results of this procedure were compared with global optima obtained using a genetic algorithm. The optima obtained by the sequential approximate optimization procedure were within 0.5% of the global optima.

Acknowledgments

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